MEASUREMENT OF THERMAL DIFFUSIVITY OF HEMISPHERICAL SAMPLES (BISMUTH)

A. A. Baskakova, V. E. Zinov'ev, and L. D. Zagrebin

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The temperature distribution in a hemispherical sample with a point source (sink) is obtained for the case where the heat is supplied in the form of a pulse of finite length. The results of measurement of the thermal diffusivity of solid and liquid bismuth are presented.

A method of measuring the thermal diffusivity of samples of hemispherical shape was described by us earlier [1]. The heat pulse was produced by a water drop at the center of a hemispherical crucible with solid or liquid metal. A sensor (thermocouple) was placed at a certain distance from the center. The thermal diffusivity of the metal was determined from the time lag of the thermocouple signal

$$a = \frac{\mathrm{Fo}_{\frac{1}{2}} R^2}{\tau_{\frac{1}{2}}}, \tag{1}$$

where $\mathrm{Fo}_{1/2}$ is the Fourier number (nondimensional time); R is the radius of the crucible; and $\tau_{1/2}$ is the time taken for the signal to reach one half of its maximum value.

The computational formulas for the thermal diffusivity were obtained from the solution of the heat conduction equation for the hemispherical sample with an instantaneous heat source (sink). It is meaningful to extend the solution of heat conduction equation to the case where the heat is supplied in the form of a pulse of length τ'. We assume that the supplied heat flux is determined by the difference of two-step functions $\eta_1(t)$ and $\eta_2(t-\tau')$:

$$q(t) = q_0 [\eta_{1(t)} - \eta_{2(t-\tau')}]. \tag{2}$$

The solution for this case is obtained by integrating the expression for the temperature over the instantaneous hemispherical source of radius r' and energy q_0 acting in a hemisphere of radius R. When there is no heat transfer to the surface (Bi = 0, this expression has the form (for example, see [3])

$$T(r,\ t) = \frac{q_0}{\rho c_p}\left\{\frac{1}{\pi R r r'} \sum_{n=1}^{\infty} \frac{\mu_n^2 R^2 + 1}{\mu_n^2 R^2} \sin \mu_n r \sin \mu_n r' \exp(-\mu_n^2 a t) + \frac{3}{2\pi R^3}\right\}. \tag{3}$$

Integrating (3) with respect to time we get

$$T(r,\ \tau) = \frac{q_0/\tau'}{\rho c_p \pi R r r'}\left\{\int_0^{\tau}\left[\sum_{n=1}^{\infty} \frac{\mu_n^2 R^2 + 1}{\mu_n^2 R^2} \sin \mu_n r \sin \mu_n r' \exp[-\mu_n^2 a(\tau - t)]\right.\right.$$
$$\left. + \frac{3}{2\pi R^3}\right] \eta_{(t)} dt - \int_{\tau'}^{\tau}\left[\sum_{n=1}^{\infty} \frac{\mu_n^2 R^2 + 1}{\mu_n^2 R^2} \sin \mu_n r \sin \mu_n r' \exp[-\mu_n^2 a(\tau - t)]\right.$$
$$\left.\left. + \frac{3}{2\pi R^3}\right] \eta_{(t-\tau')} dt\right\} = \frac{q_0/\tau'}{a \rho c_p \pi R}\left\{\frac{R^2}{r r'} \sum_{n=1}^{\infty} \frac{1 + (\mu_n R)^2}{(\mu_n R)^4} \sin \mu_n r \sin \mu_n r'\right.$$
$$\left. \times [\exp(-a\mu_n^2(\tau - \tau')) - \exp(-a\mu_n^2 \tau)] + \frac{3}{2}\frac{a\tau'}{R^2}\right\}, \tag{4}$$

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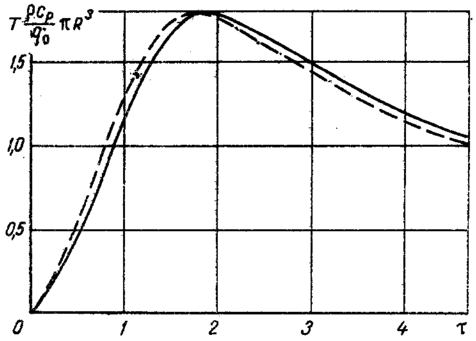

Fig. 1

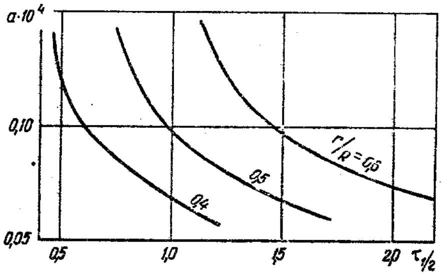

Fig. 2

Fig. 1. Time dependence of the temperature for the case when the reference signal is in the form of a pulse with finite duration: continuous line) pulse of 0.2 sec duration; dashes) instantaneous source.

Fig. 2. Dependence of the time lag of the temperature signal on the thermal diffusivity.

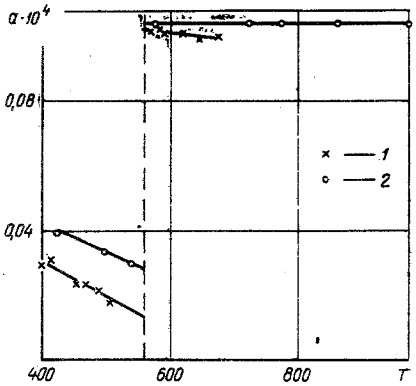

Fig. 3. Results of measurement of thermal diffusivity of bismuth: 1) our results; 2) data from [2]. $a \cdot 10^4$, m^2/sec; T, °K.

where μ_n are the positive roots of the characteristic equation

$$\mu_n R \operatorname{ctg} \mu_n R - 1 = 0. \tag{5}$$

The first six roots of Eq. (5) have been tabulated (for example, see [3, 4]).

For $\tau' \to 0$ solution (4) for the temperature distribution in the case of a finite length pulse goes over into (3); this can be shown by removing the indeterminacy in l'Hopital rule. The time dependence of the temperature was computed in accordance with (4) on Minsk-22 computer; the results are shown in Fig. 1. The following values were used in the computations: $\tau' = 0.2$ sec, $r' = 4$ mm, $r/R = 0.5$, $a = 0.15$ cm^2/sec, R = 25 mm. A comparison of this dependence with the corresponding dependence for the case of instantaneous source showed that the use of the latter gives an error of about 5% in the values of thermal diffusivity not exceeding 0.07 cm^2/sec.

The parametric dependences of a on $\tau_{1/2}$ (r/R = const), shown in Fig. 2, were constructed for computing the thermal diffusivity.

The thermal diffusivity of pure bismuth (99.98%) in solid and liquid states was investigated. The scheme of the equipment used for this investigation was similar to that described in [1]. The temperature was measured by a Chromel–Alumel thermocouple with thermal electrodes of 0.2 mm diameter. The investigated material was placed in a spherical porcelain crucible with radius R = 25 mm. The temperature signal was amplified by an I-37 amplifier and was recorded on tape of a high-response N-320 recorder. The values of $\tau_{1/2}$ were determined from the thermograms and the thermal diffusivity was determined from them using the curves shown in Fig. 2. The results thus obtained are shown in Fig. 3. The averaged data [2] are also shown in Fig. 3; the difference between these and our results does not exceed 5%. The overall error in the determination of the thermal diffusivity is made up of the errors due to the inertia of the thermocouple, the inaccuracy in the determination of the position and dimensions of the thermocouple, and also the error due to the neglect of the heat transfer to the surface. These components of the error have been estimated in [1]. The main contribution to the error came from neglecting the finite length of the pulse from the source. The use of computational equation (4), which takes account of the finite length of the pulse, permitted a significant improvement of the computations and reduced the overall error which is now about 5%.

Thus an examination of the results shows that the method of hemisphere with heat sink at the center may serve as the basis for a simple and reliable method of determining the thermal diffusivity of solid and liquid metals in a wide range of temperatures. However, for high-temperature investigations it is necessary to consider additional corrections (or their proper elimination) related to thermal radiation from an open surface.

NOTATION

T is the temperature;
q_0 is the energy of the pulse, J;
R is the radius of the crucible;
r' is the radius of the source;
c_p is the specific heat, J/kg·°K;
ρ is the density, kg/m^3;
t and τ are the times;
τ' is the pulse length;
$\tau_{1/2}$ is the time taken for the temperature signal to reach one half of the maximum value.

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